Building the Topological Tree by Recursive FCM Color Clustering

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Abstract

In this paper we define a Topological Tree *(TT) as a knowledge representation method that aims to describe important visual and spatial features of image regions, namely the* color similarity, *the* inclusion *and the* spatial adjacency. *The topological tree exhibits some interesting properties that can be exploited to extract knowledge from images for information retrieval, image understanding and diagnosis purposes. Examples of applications in dermatology are described. The TT can be constructed after segmentation, by computing the spatial relationships of regions or can be generated directly during the segmentation: to this aim we present a novel recursive fuzzy c-means (FCM) clustering algorithm based on the Principal Component Analysis of the color space. The recursive FCM proves to be effective for underlining the adjacency and inclusion property of regions.*

1. Introduction

Most of image understanding tasks require two initial stages. The first is the description of the image in terms of aggregated regions of interest and the second is a structural description of the regions and their relationships. The former is generally provided by an image segmentation technique. In the latter, the knowledge representation paradigm is more dependent of the model of the target and the application. For instance graph based representation is exploited to describe complexes scenes, to recognize objects composed by several parts or to handle content-based retrieval. Often segmentation is provided by a clustering algorithm: the goal is to divide a given image $I = \{x = (i, j), 1 \leq i \leq M_1, 1 \leq j \leq M_2\}$ into a number k of clusters; points of the same cluster are alike in the sense that share a common $f(x)$ visual property. In addition, segmentation aims to partition the image into a set of connected regions $\mathbf{R} = \{R_1, \ldots, R_k\}$ such that $\bigcup R_i = I$ and $\bigcap R_i = \varnothing$. Therefore the clustering process should be followed by an analysis of the spatial relationships between points, according with a given neighborhood system $N = \{N_x : x \in I\}$, where $N_s \subset I$ is a set containing the neighbors of x (e.g. the *8-connectivity* property).

In a graph-based representation, spatial relationships between regions, extracted by segmentation, are further exploited in order to give a global description of the image. An example is the *adjacency graph* [9], a graph $G(V, E)$ whose vertexes are the image regions ($V \equiv \mathbf{R}$) and whose arcs show the adjacency property, that is a neighborhood system at region level. We could say that R_i is *adjacent* to $R_j \Leftrightarrow \exists\, x_i \in R_i, x_j \in R_j \colon x_j \in N_{x_i}$.

In this work we aim to define a new knowledge representation method that highlights, in addition to adjacency, another topological property, i.e. *inclusion*. This property can be used to describe the fact that a region is surrounded by another disjoint region, as with the property "to be inside" defined in [1]. Many images contain objects or patterns composed by shapes included each other. Examples are satellite and isometric images, or ocean depth maps, or some medical images as the ones used in this paper. We define the *Topological Tree* (TT), a powerful knowledge representation model of the image topological structure, and we propose a recursive algorithm based on fuzzy c-means color clustering to provide, at the same time, segmentation and graph-based region description.

Since color is a very important visual feature, color clustering is widely used [3, 8, 6]. Amongst the many different approaches proposed, *fuzzy c-means* (FCM) color clustering is often adopted: an example is the work of P. Schmid [7] that proposed a fuzzy c-means segmentation over the 2D of the first two components of the Karhunen-Loève transform, or Principal Component Analysis, of the color space for skin lesion images. The only drawback of Schmid's proposal is that, lacking of spatial information, the number of classes is automatically obtained by a statistical evaluation of the histogram and fails to identify small color areas in the original image, being their pixels included in larger surrounding clusters in the histogram.

In this paper we use a basic segmentation step similar to

Shmidt's approach, but improved by means of the recursive and dichotomic clustering and included in a larger process of knowledge extraction. FCM clustering, inclusion and connectivity property are used to construct the TT of the image. In the next section we define the TT, then section 3 defines the TT construction and section 4 reports results to underline the potential benefit introduced by the TT in a dermatologic melanoma diagnosis.

2. The Topological Tree

First, we consider an "extended" set of disjoint regions $\overline{\mathbf{R}} = \mathbf{R} \cup \{R_0\}$, being R_0 a dummy region representing the external boundary of an image. Let us define the inclusion property as follows:

Definition 1 A region $R_i \in \mathbf{R}$ is included in $R_j \in \overline{\mathbf{R}} \Leftrightarrow \nexists \{x_0, \ldots, x_M\} : x_n \in N_{x_{n+1}}, x_0 \in R_0, x_M \in R_i, x_n \notin R_j, 0 \leq n < M$.

This topological feature can be exploited to define the Topological tree:

Definition 2 A *Topological Tree* (TT) is a tree (N,B) whose nodes are regions of an image, and whose branches are oriented arcs and describe the inclusion relationship between adjacent regions (parent and child): given $n_1, n_2 \in N$, $(n_1, n_2) \in B$ means that n_2 is included in n_1.

In order to use the TT to describe a segmented image we can assume that $N \equiv \overline{\mathbf{R}}$ (for this reason we will use the term node and region indifferently from now). The inclusion property is neither reflexive nor symmetric, but is transitive. This means that all the regions of a sub-tree of a region are included in that region and consequently the TT root (i.e. R_0) includes all other regions. Therefore the TT is isomorphic to a sub-graph of the adjacency graph. The topological tree is a powerful knowledge representation way, able to describe the structure of an image. Given a TT of an image

- the k number of nodes (apart from the root) indicates the number of segmented regions
- the l number of leaves indicates the number of regions without holes, that do not include other regions
- the number b of levels (i.e. the maximum number of branches needed for reaching a leaf from the root) indicates the maximum number of regions included each others
- the children of the same parents represent regions that are not adjacent (w.r.t the given neighborhood system) or that are different (w.r.t. the feature f(x) used in region segmentation).

These properties are semantically valuable since give many information on the topology of the patterns depicted in the image. The topology can be used to help the image classification or to compare two images and searching from image similarity. The TT exploitation for dermatoscopic image interpretation is outlined in the last section.

3. Topological Tree Construction

To construct the TT, as previously stated, we could exploit a known segmentation algorithm for region segmentation, compute the adjacency graph and then extract a TT cutting the adjacency arcs that do not satisfy the inclusion property. In this section we propose a color clustering algorithm that provides a focused segmentation which emphasizes not only the pixel neighborhood but also the inclusion property. We call our proposal *recursive FCM color clustering*, since is based on a recursive procedure that adopts fuzzy c-means (FCM) color clustering to divide the actual color space into clusters.

The algorithm could be applied on whichever color space, but we want to obtain results similar to the dermatologist perception of color, so we transform our images from the original RGB color space into *CIE* $L^*a^*b^*$ [5], a uniform color space in which equal distances mean almost equal perceived chromatic difference.

3.1. Karhunen-Loève Transform

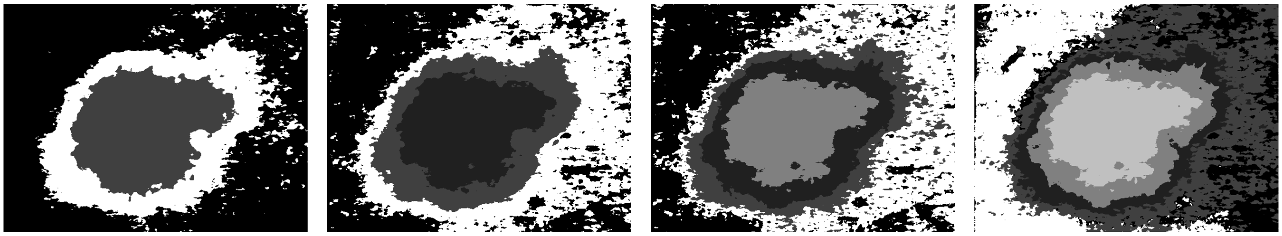
The first step is a suitable reduction of the search space. A common way used for the reduction of data, is to discard components using the Karhunen-Loève transform (KL)[4]. This transform consists in the projection of the vectors to be reduced on the eigenvectors of the covariance matrix, computed using the following equation:

$$\mathbf{C}_x = \frac{1}{M} \sum_{k=1}^{M} \left(\mathbf{x}_k \cdot \mathbf{x}_k^T - \mathbf{m}_x \cdot \mathbf{m}_x^T \right) \quad (1)$$

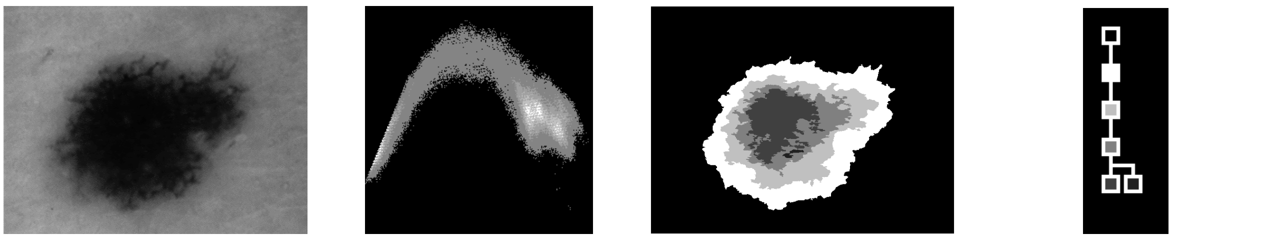
where M is the number of data samples and $\mathbf{m}_x$ is the mean vector of the image. We define a matrix $\mathbf{A}$ whose rows are the eigenvectors of matrix $\mathbf{C}$ ordered by decreasing eigenvalue. The KL transform of vector $\mathbf{x}$ is then defined as $\mathbf{y} = \mathbf{A} \cdot (\mathbf{x} - \mathbf{m}_x)$.

In our case the three components (L^*,a^*,b^*) have to be reduced to just two components and the samples are the image pixels. Processing with the KL transform does not change the uniformity of the chosen color space [7].

We then discard the third component of the transform and rescale and quantize the first two components to 256 levels in order to store each transformed pixel in a byte-type variable. From these two components a 2D histogram is computed and used for the next steps (fig.1(f)).



(a) Fuzzy c-means (c=3) (b) Fuzzy c-means (c=4) (c) Fuzzy c-means (c=5) (d) Fuzzy c-means (c=6)



(e) Original image (f) KL 2D histogram (g) TT based segmentation (h) Topologic Tree obtained

Figure 1. Comparison of multiple-region and Topological Tree color segmentation

3.2. Fuzzy c-means Clustering

The fuzzy c-means (FCM) algorithm is a robust clustering technique, especially efficient for cluster center computation. Being c the number of classes, we use the following two recurrent equations [7]:

$$U_{ik} = \left(1 + \sum_{\substack{j=1 \\ j \neq i}}^{c} \left(\frac{\|\mathbf{x}_k - \mathbf{v}_i\|^2}{\|\mathbf{x}_k - \mathbf{v}_j\|^2} \right)^{\frac{1}{m-1}} \right)^{-1} \qquad (2)$$

$$\mathbf{v}_i = \frac{\sum_{k=1}^{M} (U_{ik})^m \cdot \mathbf{x}_k}{\sum_{k=1}^{M} (U_{ik})^m} \qquad (3)$$

where U_{ik} is the fuzzy membership of $\mathbf{x}_k$to class i and $\mathbf{v}_i$ is the i^{th} class center. The weighting exponent m defines the fuzziness of the membership values.

Details of characteristics and properties of the FCM algorithm can be found in [6] and in [2].

3.3. Topological Tree

The tree construction follows a recursive procedure that starts with the analysis of the KL 2D histogram of the current set of pixels (called R in the algorithm of fig. 2). Two clusters (C1 and C2) are created: each x_k is associated to the cluster i whose U_{ik} is maximum.

In order to provide a less strict acceptance of inclusion relationship, the function testing the inclusion property is implemented with the regions convex hulls, meaning that a region is included into another if all its points belong to the convex hull of the other.

If it is possible to select an external cluster (C_{ext}) that includes the other, its connected component with maximum extent is added to the tree, while the other is further processed together with all the other non assigned components; otherwise, both clusters are recursively processed. The algorithm stops when the region to analyze is too small. Two tree operations, merging with ancestor or being a node on itself, are defined for these regions depending on size. The former accounts for little negligible spots (T_{stop}), while the latter define the smallest region that can be further processed (T_{split}). Recursively we obtain the topological tree of the regions, whose pseudo-code is shown in fig. 2.

4. Application and Results

The algorithm has been tested on epiluminescence images, obtained with a dermatoscopic microscope and digitized to 640x480, 16 bit per pixel color images. The database used as workbench has around 600 images associated with their diagnosis.

The first observation is that our recursive approach allows a focused segmentation of region of interest, achieving a more meaningful analysis. The results of TT construction are shown in figure 1(g), while the corresponding segmentations, obtained with a standard fuzzy-c means, are in figures 1(a) to 1(d): it is possible to note that increasing the number

```
node AnalyzeSet (SetOfPixels R, node N)
{
  for each C in Regions(R) {
    if (Area(C)<Tstop) {
      Merge (C,N);
      return;
    }
    if (Area(C)<Tsplit) {
      AddNodeToTree (C,N);
      return;
    }
    [C1,C2] = FCM (C);
    [Cint,Cext] = VerifyInclusion ([C1,C2]);
    if (exists([Cint,Cext])) {
      Cmax = MaxExtentConnectedComp (Cext);
      Nnew = AddNodeToTree (Cmax,N);
      AnalyzeRegion (C-Cmax,Nnew);
    }
    else {
      AnalyzeRegion (C1,N);
      AnalyzeRegion (C2,N);
    }
  }
}
```

Figure 2. Pseudo-code of the algorithm

of clusters without any spatial constraint does not provide a better segmentation of the lesion compared to the one obtained with recursive FCM 1(g). A peculiar characteristic of our segmentation is that the stop condition is controlled by features of the region under examination, without relationship with the rest of the image, while the standard criterion applied in clustering segmentation is the cluster number, that has to be a priori fixed. Indeed, the main contribution of the work is the knowledge representation provided by the algorithm, in the form of a topological tree. Different aspects of the image, significant for melanoma diagnosis, can be examined directly working on the TT and tuning the stop conditions: we can point out the general structure, or we can focus on details such as black dots. Examples are shown in figure 3, where different cases and different TT depths are illustrated. The first two examples show two coarse segmentations (with a "high" *Tstop* threshold): the very different topological structure is embodied in the very different TT structure. Instead, the fine segmentation in the third row allows also a quantitative evaluation: the number of black dots, an important lesion feature, can be computed by counting the number of leaves of the TT.

5. Conclusions

The proposed algorithm provides an interesting knowledge representation of the spatial relationship between image regions and it is exploited as a knowledge extraction method to support the dermatological diagnosis. A qualitative evaluation of the dermatologists shows an interesting concordance between the complexity of the tree structure and the malignancy of the lesion. In the future, the analysis of lesion structure could be automatically provided by TT only; moreover TT could be used to search for similarities in the medical database.

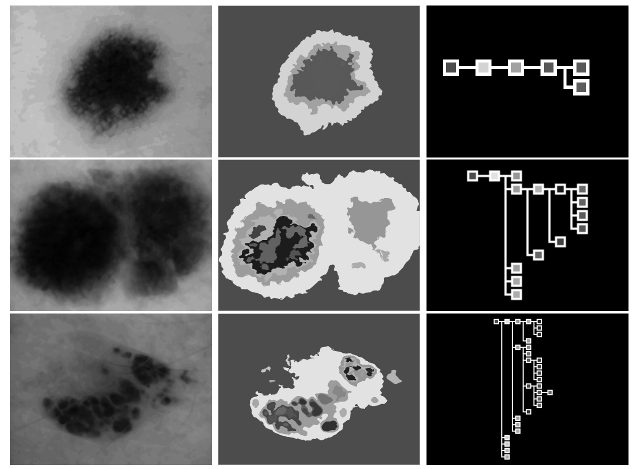


Figure 3. Examples of TT segmentation

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